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### Fatigue Crack Growth Rates in Adhesive Joints Tested at Different Frequencies

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# Fatigue Crack Growth Rates in Adhesive Joints Tested at Different Frequencies\*

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Mode I fatigue crack growth tests were conducted on joints bonded with a filled adhesive (A) at 20 Hz and 2 Hz and on joints bonded with a filled and toughened adhesive (B) at 20 Hz, 2 Hz, 0.2 Hz and 0.02 Hz. Strain energy release rate,  $G$ , and  $J$ -integral were evaluated based on elastic and elastoplastic finite element analyses (FEA) of the joints bonded with adhesive A and B, respectively. For the configurations considered,  $J$  was found to be path-independent and did not differ much from  $G$ . The fatigue crack growth rate (FCGR),  $da/dN$ , in the joints bonded with adhesive A was relatively independent of frequency while it increased with decreasing frequency at given  $\Delta G$  for the joints bonded with adhesive B. The fatigue processes in both adhesives involved the cracking of the filler particles and subsequent linkage of the resultant microcracks. The process zone in adhesive B is larger than that in adhesive A and it increases with decreasing frequency. It is suggested that this variation in process zone size can account for the observed fatigue behaviour. The fatigue crack growth velocity,  $da/dt$ , was also calculated for the joints bonded with adhesive B and the variation of  $da/dt$  with test frequency at given  $\Delta G$  is much smaller than the variation in  $da/dN$ , suggesting a creep effect in the fatigue crack growth.

**KEY WORDS:** Adhesive joint fatigue; frequency dependent;  $J$  integral; filler particle cracking; bondline thickness effects; finite element analysis.

## INTRODUCTION

Many adhesive joints are subject to cyclic loads during service, e.g. bonded bridge strengthening plates and aircraft fuselages.<sup>1</sup> Since adhesive joints are less tolerant of cyclic than of static loading,<sup>2</sup> much research work has been carried out on various aspects of the fatigue behaviour of adhesive joints. This includes the correlation of

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fracture mechanics parameters with fatigue crack growth rates,<sup>3,4</sup> fatigue micromechanisms<sup>5,6</sup> and mixed mode effects.<sup>7-11</sup>

The time span of each fatigue load cycle varies significantly from structure to structure. It could be, for instance, several seconds in the case of bridge plates to tens of hours in the case of aircraft fuselages. In reality, various fatigue frequencies could be encountered in service. Therefore, frequency effects on joint performance have been considered by a number of investigators. For example, Marceau, McMillan and Scardino<sup>12</sup> studied fatigue crack growth rates in both lap-shear and double cantilever beam joints at frequencies ranging from  $2 \times 10^{-4}$  Hz to 30 Hz and found low frequency fatigue more damaging to adhesive joints. Althof<sup>13</sup> studied the build up of shear strain in adhesive joints during fatigue loading at frequencies ranging from  $2.8 \times 10^{-4}$  Hz to  $8.3 \times 10^{-3}$  Hz and suggested that the fatigue failure is creep controlled. Following this work, Luckyram and Vardy<sup>14</sup> studied the fatigue performance of two toughened epoxy adhesives and found no frequency effect on fatigue behaviour in the frequency range of 0.5 Hz to 5 Hz.

This current work deals with the effects of adhesive mechanical properties and microstructure on FCGR (fatigue crack growth rate) in steel-steel joints bonded with two commercial adhesives, A and B. Mechanical data for these adhesives are presented in the next section. Of particular interest are the effects of the filler particles (added to the base epoxy matrix of both adhesives) and rubber (present in adhesive B) on the fatigue crack growth behaviour. Further details of these filler particles are given in the Experimental Results and Discussion section. To study any frequency effect on fatigue crack growth rates, fatigue tests were carried out over the range of 0.02 Hz to 20 Hz. Finite element analysis (FEA) has been carried out to analyse the stress/strain distributions in the joints and to evaluate two parameters which characterise crack driving force,  $J$ -integral and strain energy release rate,  $G$ , allowing for the frequency dependence of the tensile moduli of the adhesives and the corresponding elastoplastic behaviour. Although the  $J$ -integral has been proven to be path-independent in homogenous materials,<sup>15</sup> it was thought necessary to check its path independence in adhesive joints for its possible use as a crack driving force in joints bonded with elastoplastic adhesives. As adhesive B was found to be rate-sensitive, preliminary studies concerning the effect of creep in the fatigue crack growth process has been carried out.

## EXPERIMENTAL PROCEDURES

### a) Material Data and Joint Configuration

From the quasi-static T-peel and lap shear tests conducted it was found that adhesive B has a higher T-Peel strength but a lower shear strength than does adhesive A. In addition, the bulk tensile stress-strain behaviour of adhesive B exhibits a strong dependence on strain rate while that of adhesive A does not. This can be seen in Figure 1, the data for which has been obtained<sup>16</sup> by carrying out tensile tests at varying strain rates on flat "dog-bone" specimens of the bulk adhesive. A summary of the tensile moduli derived from these stress-strain curves is given in Table 1.

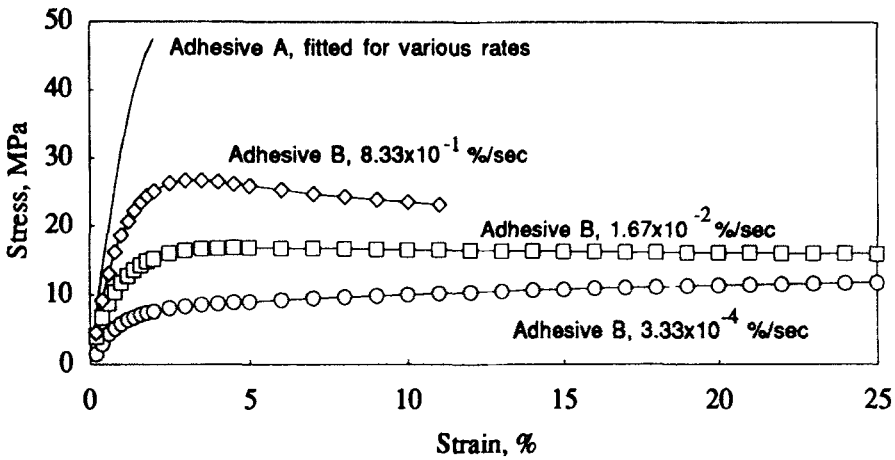


FIGURE 1 Tensile stress-strain curves of adhesives A and B as a function of strain rate.<sup>16</sup>

TABLE 1  
Tensile modulus of adhesive A and B

Adhesive	Strain rate (%/sec)	Tensile Modulus (MPa)
A	$1.67 \times 10^{-2}$	3400
B	$3.33 \times 10^{-4}$	390
B	$1.67 \times 10^{-2}$	1060
B	$8.33 \times 10^{-1}$	2630

The geometry of the fatigue test specimens is shown in Figure 2. Joints were made with bondline thickness of both 0.2mm and 1 mm. The steel substrates were bead blasted, ultrasonically cleaned and treated with silane, prior to bonding. The adhesive was dispensed from a twin pack cartridge system through a mixing nozzle directly onto

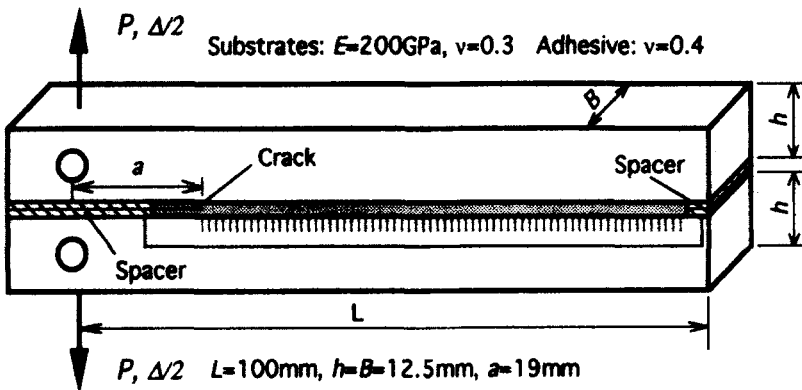


FIGURE 2 Adhesive joint test geometry.

one of the substrates. The joint was then assembled, controlling the adhesive thickness through spacers at either end of the joint and forming a pre-crack using thin polyimide film. The bonded specimen was then set in a jig and cured at 40°C for 24 h. Further manufacturing details have been given elsewhere.<sup>6</sup> The cured specimens were stored in a desiccator before testing. Figure 2 shows the dimensions of the finished specimens.

### b) Fatigue Testing

Fatigue tests were carried out using an Instron model 1341 servo-hydraulic machine under load control. Except for a threshold investigation in the joint bonded with adhesive A, all tests were done under a constant load amplitude ratio of 0.2 using a sinusoidal wave form. During testing, the ambient temperature was kept at around 20°C. The joints bonded with adhesive A were tested at frequencies of 2 Hz and 20 Hz and those bonded with adhesive B were tested at frequencies of 0.02 Hz, 0.2 Hz, 2 Hz and 20 Hz. Those joints with a bondline thickness of 1 mm were tested at 2 Hz only. A scale with 0.1 mm-spaced gridlines was attached to the lower substrate parallel to the adhesive line as shown in Figure 2. A travelling microscope was used to measure the crack length.

## DETERMINATION OF $J$ AND $G$ USING FINITE ELEMENT ANALYSIS

A plane strain model based on 8-noded quadrilateral elements was generated for the linear finite element analyses (FEA). As all the fatigue cracks propagated cohesively, only the upper half of the joint was modelled. The 0.2 mm thick adhesive bondline was divided into four layers around the crack tip. A more refined mesh with eight layers was analysed and no significant difference was found in the results for the strain energy release rate which was determined using a virtual crack closure method. This appears to be broadly in agreement with work by Schmueser *et al.*<sup>17</sup> who used five four-noded elements across a similar size bondline in conjunction with a crack closure method. Further away from the tip, the adhesive elements change gradually to rectangles with an aspect ratio of 5 to 1. Details of the finite element mesh are shown in Figure 3. The mesh used to analyse the joint with an adhesive thickness of 1 mm was broadly similar but utilised 14 layers of elements across the adhesive at the crack tip. Different crack lengths were realised by detaching elements from one end of the model and attaching them to the other. The FEA was run using the ANSYS finite element code.

Elastoplastic FEA were carried out for the joints bonded with adhesive B loaded quasi-statically. The mesh used for these analyses had a little more refinement around the crack tip but was otherwise the same as that discussed above. Rate-dependent material behaviour has been incorporated by evaluating, in an iterative manner, an effective strain rate which increases with increasing fatigue frequency. The details of this approach can be found in our previous work.<sup>6</sup> The von Mises yield criterion was adopted for the elastoplastic analyses in which loads were applied in steps, each including several sub-steps. The Newton-Raphson equilibrium iteration procedure was used to obtain convergence at the end of each load sub-step. To investigate the difference between  $J$  and  $G$ , a load of 1025 N, the typical maximum value achieved in

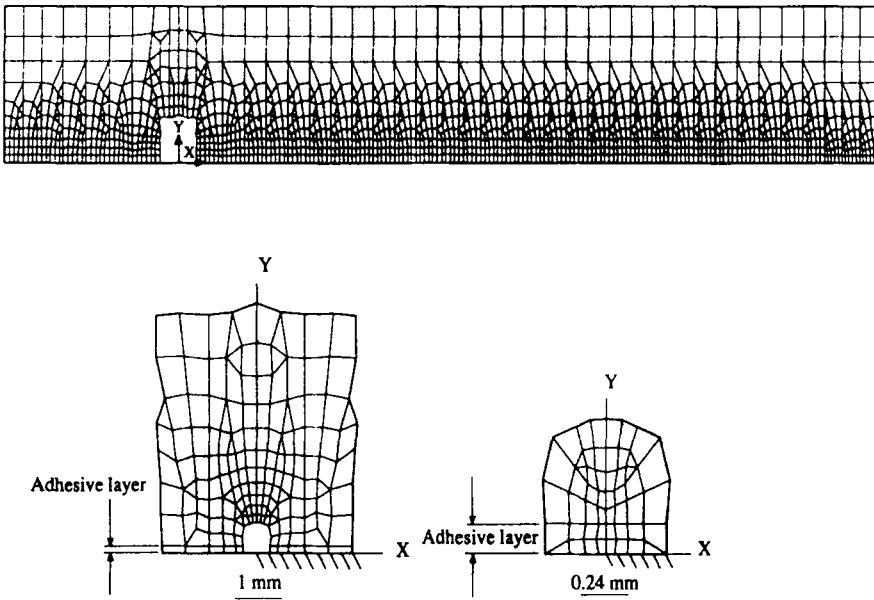


FIGURE 3 Details of the finite element mesh.

quasi-static tests, was chosen for the FEA. The results of the elastoplastic FEA show that there is significant plastic deformation within the adhesive layer, especially at low strain rates. Figure 4 shows the plastic strain distribution around the crack tip area at a strain rate of  $8.33 \times 10^{-4} \%$ /sec. The plastic zone is restricted to the adhesive layer and is elongated in the bondline direction. At this strain rate, the plastic zone stretches 3.8 mm from the crack tip, a distance 19 times the thickness of the adhesive layer. In contrast, the plastic zone only extends about 0.3 mm or 1.5 times the thickness of the adhesive layer behind the crack tip.

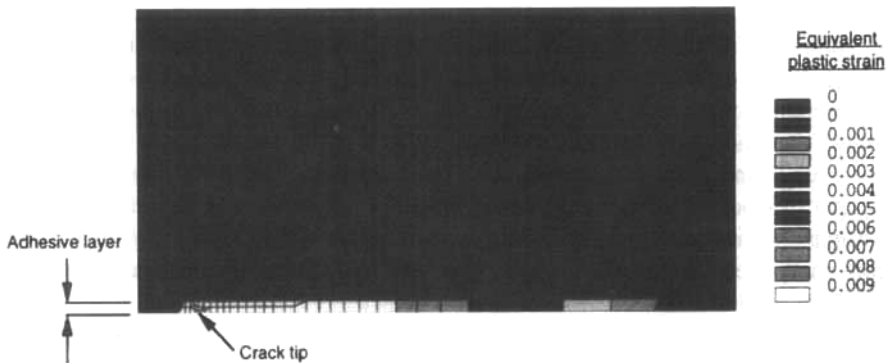


FIGURE 4 Elongated plastic zone in the joint bonded with adhesive B.

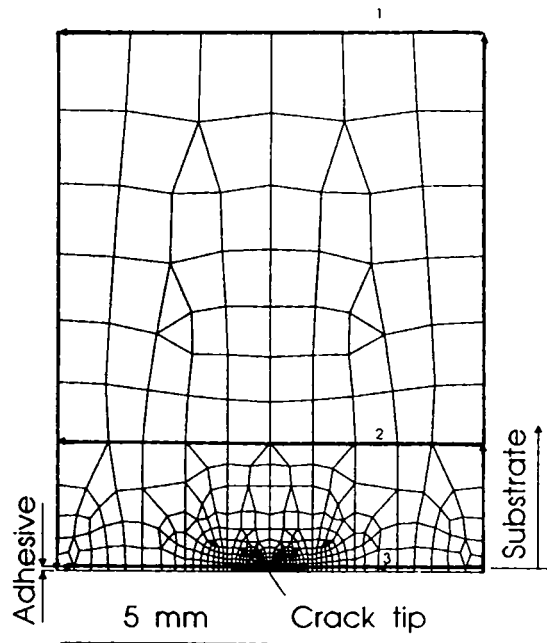


FIGURE 5 Three contours for  $J$ -integral evaluation.

The expression for  $J$  in 2-dimensional form is:

$$J = \int_{\Gamma} w dy - \int_{\Gamma} \left( t_x \frac{\partial u_x}{\partial x} + t_y \frac{\partial u_y}{\partial x} \right) ds \quad (1)$$

$\Gamma$  is any path surrounding the crack tip;  $w$  is the strain energy density;  $t_x$  is the traction vector along  $x$  axis and is equal to  $(\sigma_x n_x + \sigma_{xy} n_y)$ ;  $t_y$  is the traction vector along the  $y$  axis and is equal to  $(\sigma_y n_y + \sigma_{xy} n_x)$ ;  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_{xy}$  are the stress components;  $n$  is the unit outward normal vector to the path  $\Gamma$ ; and  $s$  is the distance along the path  $\Gamma$ .

The path independence of the  $J$ -integral has been established for homogeneous materials<sup>15</sup> and for heterogeneous welded joints.<sup>18</sup> To check the path-independence of the  $J$ -integral for the joint which contains very dissimilar materials (i.e. polymeric adhesive and steel substrate in this work), three contours (paths), shown in Figure 5, were chosen along which the  $J$ -integral was evaluated. The first one (contour 1) cuts through the whole substrate and incorporates part of the free surface of the substrate. The second one (contour 2) runs into the substrate. The third one (contour 3) incorporates part of the interfaces between the adhesive and the substrate. The elastoplastic  $J$ -integrals were evaluated at a strain rate of  $8.33 \times 10^{-4}$  %/sec. Values of strain energy release rate,  $G$ , were also obtained by running an elastic analysis at the same load level of 1025 N.

The evaluated  $J$ -integrals are listed in Table II which shows that the  $J$ -integral is path-independent in the adhesive joint. Although there is significant yielding in the adhesive layer as shown in Figure 4, the average  $J$ -integral is only marginally higher

TABLE II  
Values of  $J$ -integral and  $G$

Contour	1	2	3
$J_{e/p}$ , N/m	189(189)	192(189)	187(189)
$J_e$ , N/m	186(187)	188(187)	185(187)
$G$ , N/m		185	

Note:  $J_{e/p}$  and  $J_e$  represent  $J$ -integrals evaluated by elastoplastic FEA and linear elastic FEA, respectively. The values in brackets are the average of the  $J$ -integrals obtained along the three contours.

than  $G$ . Therefore, it was thought reasonable and convenient to use  $G$  (obtained from a linear analysis) as a characterising parameter for the crack driving force for the joints bonded with both adhesives. Since the modulus of adhesive A is almost independent of strain rate, a value of 3500 MPa obtained from the tensile test was used. The moduli used for adhesive B, which were evaluated in an iterative manner,<sup>6</sup> were 2370 MPa, 1390 MPa, 920 MPa, and 640 MPa corresponding to cycling frequencies of 20 Hz, 2 Hz, 0.2 Hz and 0.02 Hz. Using these data, the strain energy release rate,  $G$ , could be found for any fatigue load as a function of crack length and fatigue frequency.

## EXPERIMENTAL RESULTS AND DISCUSSION

Figure 6 shows the log-log plots of FCGR,  $da/dN$ , versus  $\Delta G$ , for the joints with a bondline thickness of 0.2 mm, together with regression lines obtained based on a Paris relation, i.e.  $da/dN = k\Delta G^m$ , where  $k$  and  $m$  are constants. Since the fatigue crack growth rate data for the joints bonded with adhesive A are essentially the same at 2 Hz and 20 Hz, the data, excluding those near the threshold, have been fitted to a single line. Figure 7 and 8 show the typical fatigue fracture surfaces for the joints bonded with adhesive A and B, respectively. The fracture surface did not appear to change significantly with test frequency and was cohesive down the centre of the adhesive layer for all joints tested. During testing, microcracks were seen to initiate and propagate ahead of the major crack as has been discussed in our earlier work (Xu, Crocombe and Smith,<sup>6</sup>). The microcracks propagated both forward and backward and eventually linked with the major crack. Compared with the joints bonded with adhesive A, there is a far more extensive damage zone in the bondline of adhesive B, a feature supported by the results of the elastoplastic FEA. Sometimes two or even three parallel microcracks were seen to propagate at the same time. However, one of the cracks gradually dominated and the others stopped growing. Based on such observations, it is suggested that the fatigue crack growth process is one of repeated microcracking and subsequent linkage of these microcracks with the major crack.

Flat facets are seen in Figures 7 and 8 and energy spectrum analysis showed that in both adhesives the flat facets are broken filler particles rich in magnesium silicate.<sup>6</sup> In addition to the facets, voids (presumably associated with debonded particles) are observed, in particular on fracture surfaces of the joints bonded with adhesive A, Figure 7. The size of the voids is generally smaller than that of the facets. The particles



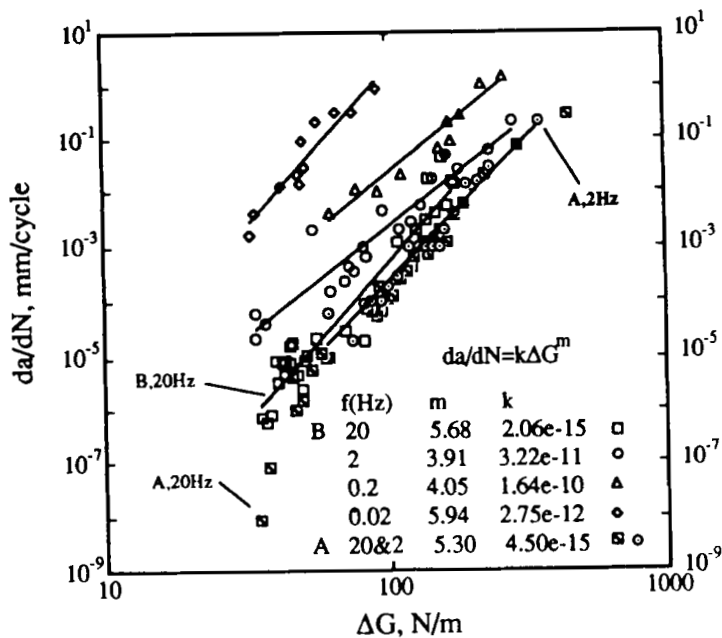


FIGURE 6 FCGR of joints bonded with adhesives A and B.

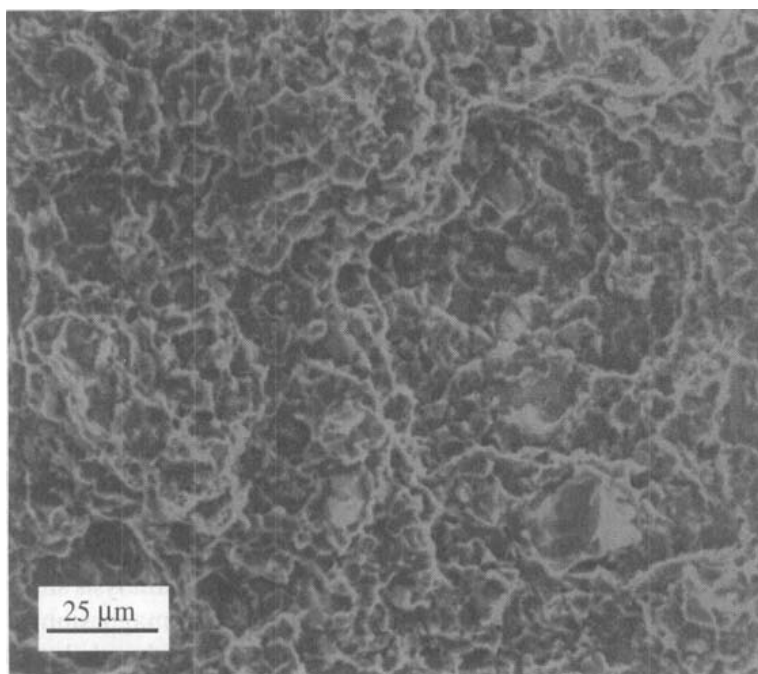


FIGURE 7 Fracture surface (SEM) of joint bonded with adhesive A.

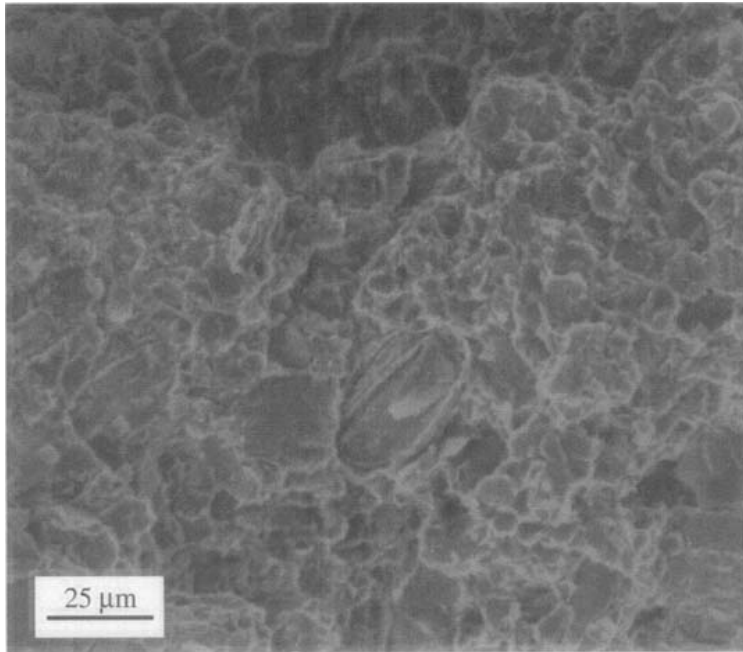


FIGURE 8 Fracture surface (SEM) of joint with adhesive B.

which debonded and resulted in voids contain no magnesium but have a high calcium content. The major fatigue crack growth mechanism can, thus, be proposed as the cracking of large filler particles and subsequent linkage of the microcracks so formed with the major crack, Figure 9.

It is clear that at 2 Hz and 20 Hz, and at any given value of  $\Delta G$ , the FCGR is higher in the joints bonded with adhesive B which is rubber-toughened. This may be due to the lower strength of this adhesive and the slight difference in fatigue crack growth mechanism. As mentioned already, the small calcium-rich particles in adhesive A contributed to microcrack initiation by void nucleation. This void growth mechanism may require more energy than particle cleavage, provided that the total volume fraction of particles is comparable with that in adhesive B. The higher fatigue crack growth per cycle in the joints bonded with adhesive B demonstrates that a rubber-toughened matrix does not guarantee a better fatigue performance.

There is a clear effect of frequency on FCGR for joints bonded with adhesive B. At a given  $\Delta G$ , the fatigue crack growth per cycle is highest at 0.02 Hz and lowest at 20 Hz, Figure 6. SEM examination of the fracture surface is not very helpful in clarifying the cause of this effect because the mechanism of crack growth does not appear to change with frequency. The explanation might lie in the reduction in the yield strength of adhesive B with strain rate, Figure 1. The fatigue failure mechanism involves microcracking ahead of the major crack. From Figure 4 it is reasonable to argue that the microcracking in the adhesive will happen only in an elongated plastic zone, the length of which should be inversely proportional to the square of the yield strength of the adhesive, Figure 10. Therefore the plastic zone in adhesive B at 0.02 Hz is the longest and, hence, the fatigue crack grows most quickly.

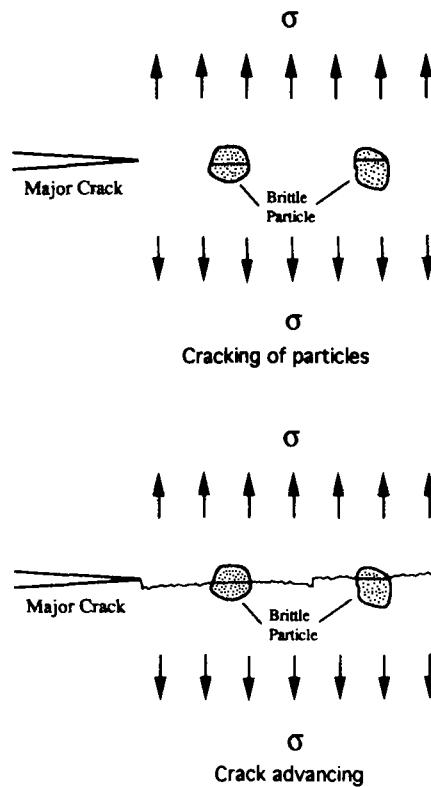


FIGURE 9 Schematic representation of microcracking and linkage.

The above argument based on an elongated plastic zone implies that: a) if the restriction by the substrates is relaxed, e.g. by increasing the bondline thickness, the fatigue crack growth rate will tend to approach that of the bulk adhesive; b) once the restriction is removed, the FCGR will decrease due to the reduction in the scale of the plastic zone in the direction of crack growth. Measurements of fatigue crack growth in the joints with a bondline thickness of 1 mm and CT specimens (50 mm × 48 mm × 6 mm) of bulk adhesives A and B carried out at 2 Hz have confirmed this hypothesis. The FCGRs in the bulk adhesives are compared with those of the joints bonded with adhesives A and B in Figures 11 and 12, respectively. The figures show that at a given  $\Delta G$ , FCGR is highest in the joints with 0.2 mm thick adhesive bondline and lowest in the bulk adhesive, with the data for the 1 mm thick bondline lying between the two. Increasing the adhesive bondline thickness (in the case of bulk adhesive, the bondline is taken as infinite) is more effective in reducing the fatigue crack growth rate for the joints bonded with adhesive B. This is expected because being a more ductile material, adhesive B is more capable of extending its plastic zone in the presence of constraining substrates. The failure surfaces of the thicker joint and bulk CT specimen were essentially the same as that found for the 0.2 mm joints and shown earlier in Figures 7 and 8 for adhesives A and B, respectively.

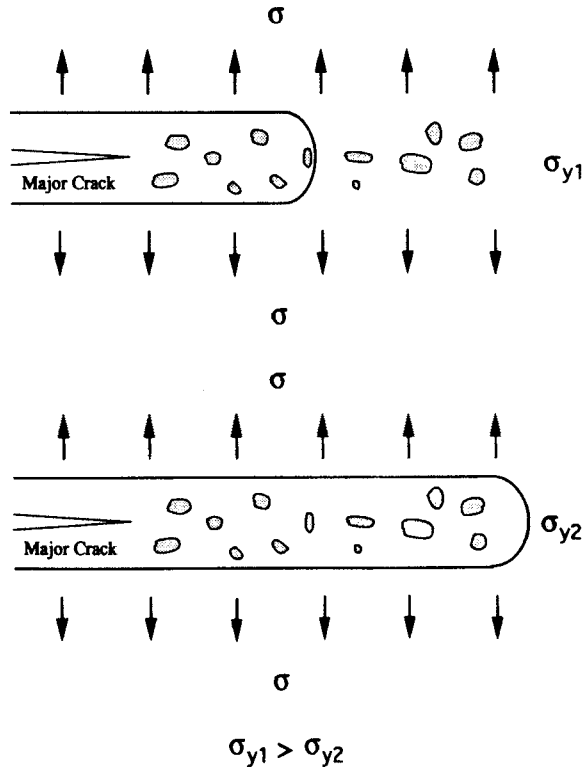


FIGURE 10 Process zone size as a function of yield stress.

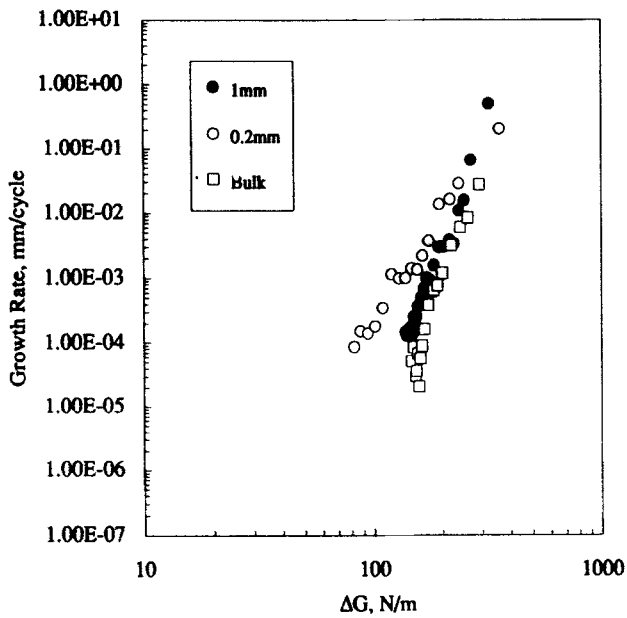


FIGURE 11 Fatigue crack growth rates in bulk adhesive, joints with 0.2 mm and 1 mm bondline thickness (adhesive A, 2 Hz).

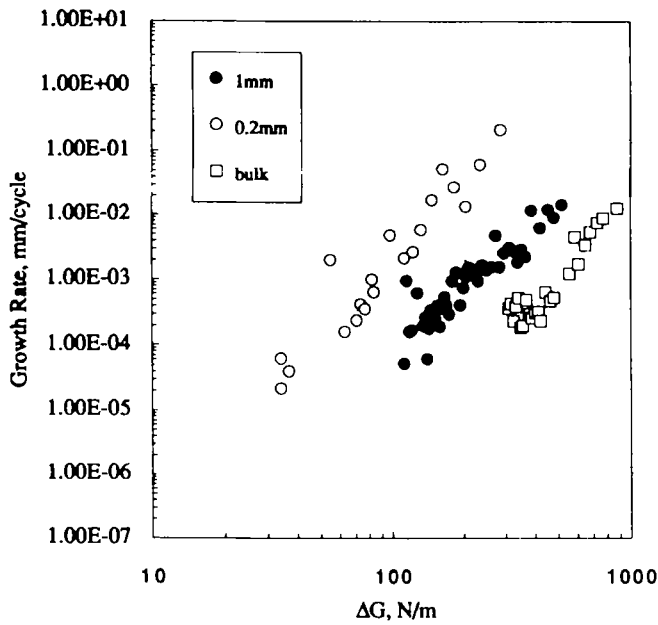


FIGURE 12 Fatigue crack growth rates in bulk adhesive, joints with 0.2 mm and 1 mm bondline thickness (adhesive B, 2 Hz).

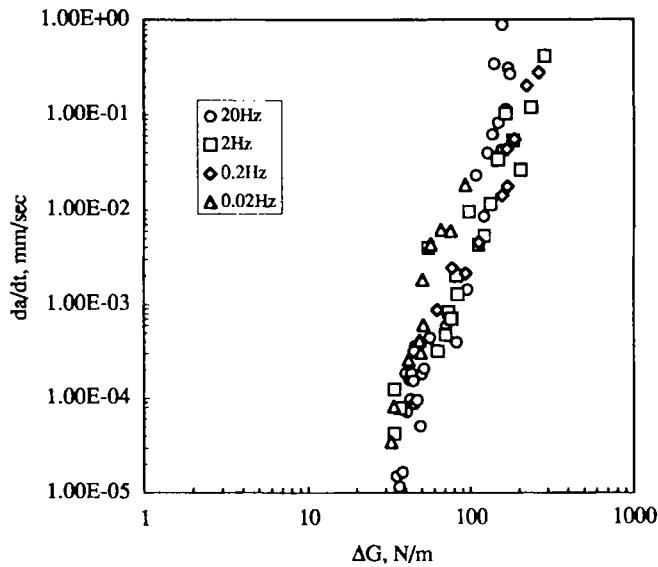


FIGURE 13 Fatigue crack velocity in joint bonded with adhesive B.

Finally, in an attempt to bring together the separate  $da/dN$  vs  $\Delta G$  curves of the joints bonded with adhesive B, the fatigue crack growth rates,  $da/dN$ , were converted into fatigue crack growth velocity,  $da/dt$ . Figure 13 shows the relation between  $da/dt$  and  $\Delta G$  for the joints bonded with adhesive B at the four fatigue frequencies. It is seen that at a given  $\Delta G$ , the four sets of  $da/dt$  data are much closer to each other than the  $da/dN$  data, with minimal frequency effect being demonstrated. This suggests that a time-dependent factor, such as creep deformation, could also have played a role in the fatigue crack growth in the joints bonded with adhesive B. This subject is currently being investigated further.

### CONCLUDING REMARKS

Finite element analyses have been carried out based on both linear elastic and elastoplastic stress-strain relations in order to evaluate  $G$  and  $J$  in the mode I adhesive joints. The elastoplastic FEA shows that at low strain rates there is extensive plastic deformation in the form of an elongated plastic zone ahead of the crack tip. As in homogeneous materials, the  $J$ -integral remains path-independent in the adhesive joints. Even in the presence of the extensive plasticity in the adhesive layer, the strain energy release rate,  $G$ , is almost coincident with  $J$ -integral and therefore  $G$  is a good characterising parameter for the crack driving force in the adhesive joints. The fatigue crack growth in the joints is a process of microcrack formation and subsequent linkage of these microcracks with the main crack. The fatigue crack growth rate increases as frequency decreases for the joints bonded with a filled and toughened adhesive. This has been attributed to the variation of strength of the adhesive with strain rate, resulting in a different process zone size which increases as the fatigue frequency decreases, leading to an increasing fatigue crack growth per cycle,  $da/dN$ . Decreasing the length of the process zone by testing joints with larger adhesive thickness and bulk specimens has led to a reduction of FCGR. The fatigue crack growth velocities in terms of  $da/dt$  do not appear to differ greatly for the joints bonded with adhesive B. This might indicate a significant creep effect in the fatigue crack growth.

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